

Relations & Functions

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If $R = \{(x, y) : x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation in set Z , then domain of R is
(a) $\{0, 1, 2\}$ (b) $\{-2, -1, 0, 1, 2\}$ (c) $\{0, -1, -2\}$ (d) $\{-1, 0, 1\}$
Ans: (b) $\{-2, -1, 0, 1, 2\}$
Let $y = 0$, then $x^2 \leq 4 \Rightarrow x = 0, \pm 1, \pm 2$
Thus, domain of $R = \{-2, -1, 0, 1, 2\}$
2. Let the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$. Then $[1]$, the equivalence class containing 1, is :
(a) $\{1, 5, 9\}$ (b) $\{0, 1, 2, 5\}$ (c) ϕ (d) A
Ans: (a) $\{1, 5, 9\}$
3. Given triangles with sides $T_1 : 3, 4, 5$; $T_2 : 5, 12, 13$; $T_3 : 6, 8, 10$; $T_4 : 4, 7, 9$ and a relation R in set of triangles defined as $R = \{(\Delta_1, \Delta_2) : \Delta_1 \text{ is similar to } \Delta_2\}$. Which triangles belong to the same equivalence class?
(a) T_1 and T_2 (b) T_2 and T_3 (c) T_1 and T_3 (d) T_1 and T_4
Ans: (c) T_1 and T_3
 T_1 and T_3 are similar as their sides are proportional.
4. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A ?
(a) $(1, 1)$ (b) $(1, 2)$ (c) $(2, 2)$ (d) $(3, 3)$
Ans: (b) $(1, 2)$
5. Given set $A = \{1, 2, 3\}$ and a relation $R = \{(1, 2), (2, 1)\}$, the relation R will be
(a) reflexive if $(1, 1)$ is added (b) symmetric if $(2, 3)$ is added
(c) transitive if $(1, 1)$ is added (d) symmetric if $(3, 2)$ is added
Ans: (c) transitive if $(1, 1)$ is added
Here $(1, 2) \in R, (2, 1) \in R$, if transitive $(1, 1)$ should belong to R .
6. Let ' f ' : $R - \{2\} \rightarrow R - \{1\}$ be a function defined by $f(x) = \frac{x-1}{x-2}$, then ' f ' is
(a) into function (b) many one function
(c) bijective function (d) many one, into function.
Ans: (c) bijective function

CBSE ACADEMY PLUS

7. Let the function 'f' : $N \rightarrow N$ be defined by $f(x) = 2x + 3$, $x \in N$. Then 'f' is
(a) not onto (b) bijective function
(c) many-one, into function (d) None of these

Ans : (a) not onto

$$\text{if } y = f(x) = 8 \Rightarrow 8 = 2x + 3 \Rightarrow x = 5/2 \notin N.$$

Hence, not onto.

8. Set A has 3 elements and the set B has 4 elements. Then the number of injective functions that can be defined from set A to set B is
(a) 144 (b) 12 (c) 24 (d) 64

Ans: (c) 24

$$\text{Total injective mappings/functions} = {}^4P_3 = 4! = 24.$$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** Given set $A = \{1, 2, 3, \dots, 9\}$ and relation R in set $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$, be an equivalence relation. The ordered pair $(1, 3)$ belongs to equivalence class related to $[(5, 3)]$

Reason (R): Any ordered pair of $A \times A$ belongs to equivalence class $[(5, 3)]$ if $(x, y) R (5, 3) \forall (x, y) \in A \times A$.

Ans: (a) Both A and R are true and R is the correct explanation of A.

10. **Assertion (A):** Let R be the relation on the set of integers Z given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.

Reason (R): A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Ans: (a) Both A and R are true and R is the correct explanation of A.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Ans: Let $A = \{1, 2, 3, 4, 5, 6\}$.

A relation R is defined on set A as:

$$R = \{(a, b) : b = a + 1\}$$

$$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$$

We observe, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$

We can say $(a, a) \notin R$, where $a \in A$.

R is not reflexive.

It can be observed that $(2, 3) \in R$, but $(3, 2) \notin R$.

R is not symmetric.

Now, $(2, 3), (3, 4) \in R$ but $(2, 4) \notin R$

As $(x, y) \in R, (y, z) \in R \not\Rightarrow (x, z) \in R$

R is not transitive

We observe, R is neither reflexive, nor symmetric, nor transitive.

12. Write the inverse relation corresponding to the relation R given by $R = \{(x, y) : x \in N, x < 5, y = 3\}$. Also write the domain and range of inverse relation.

CBSE ACADEMY PLUS

Ans: Given, $R = \{(x, y) : x \in \mathbb{N}, x < 5, y = 3\}$

$\Rightarrow R = \{(1, 3), (2, 3), (3, 3), (4, 3)\}$

Hence, required inverse relation is

$R^{-1} = \{(3, 1), (3, 2), (3, 3), (3, 4)\}$

\therefore Domain of $R^{-1} = \{3\}$ and

Range of $R^{-1} = \{1, 2, 3, 4\}$

13. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b) : a, b \in R \text{ and } a \leq b^2\}$ is neither reflexive, nor symmetric, nor transitive.

Ans: Given $S = \{(a, b) \in R \mid a \leq b^2\}$

We can consider counter example.

For reflexive: Let $\left(\frac{1}{2}, \frac{1}{2}\right) \in S \Rightarrow \frac{1}{2} \leq \left(\frac{1}{2}\right)^2 \Rightarrow \frac{1}{2} \leq \frac{1}{4}$, false, Hence, not reflexive.

For symmetric: Let $(-1, 2) \in S \Rightarrow -1 \leq (2)^2 \Rightarrow -1 \leq 4$ true,

If symmetric then $(2, -1) \in S$

$\Rightarrow 2 \leq (-1)^2 \Rightarrow 2 \leq 1$, false, Hence, not symmetric.

For transitive: Let $(25, 3) \in S$ and $(3, 2) \in S$

$\Rightarrow 25 \leq (3)^2$ and $3 \leq (2)^2 \Rightarrow 25 \leq 9$ and $3 \leq 4$, false in first and true in second case.

Moreover, If transitive then $(25, 2) \in S \Rightarrow 25 \leq (2)^2 \Rightarrow 25 \leq 4$, false

Hence, not transitive.

14. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by

$$f(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

For all $n \in \mathbb{N}$, state whether the function f is bijective. Justify your answer.

Ans: Given, $f(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

Let $x_1 = 1$ and $x_2 = 2$ be two elements of \mathbb{N} .

$\therefore f(x_1) = f(1) = (1+1)/2 = 2/2 = 1$

and $f(x_2) = f(2) = 2/2 = 1$

$f(x_1) = f(x_2)$ for $x_1 \neq x_2$

$f : \mathbb{N} \rightarrow \mathbb{N}$ is not one-one.

\Rightarrow As f is not one-one. f is not a bijective function.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Show that the modulus function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x , if x is positive or 0 and $|x|$ is $-x$, if x is negative.

Ans: $f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

One-one: Let $x_1 = 1, x_2 = -1$ be two elements belongs to \mathbb{R}

$f(x_1) = f(1) = |1|$ and $f(x_2) = f(-1) = -(-1) = 1$

$\Rightarrow f(x_1) = f(x_2)$ for $x_1 \neq x_2$

$\Rightarrow f(x)$ is not one-one.

Onto: Let $f(x) = -1 \Rightarrow |x| = -1 \in \mathbb{R}$, which is not possible.

$\Rightarrow f(x)$ is not onto.

CBSE ACADEMY PLUS

Hence, f is neither one-one nor onto function.

16. Given a non-empty set X , define the relation R in $P(X)$ as follows:

For $A, B \in P(X)$, $(A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive and not symmetric.

Ans: Let $A \in P(X)$. Then $A \subset A$

$\Rightarrow (A, A) \in R$

Hence, R is reflexive.

Let $A, B, C \in P(X)$ such that $(A, B), (B, C) \in R$

$\Rightarrow A \subset B, B \subset C$

$\Rightarrow A \subset C$

$\Rightarrow (A, C) \in R$

Hence, R is transitive.

$\emptyset, X \in P(X)$ such that $\emptyset \subset X$. Hence, $(\emptyset, X) \in R$. But, $X \not\subset \emptyset$, which implies that $(X, \emptyset) \notin R$.

Thus, R is not symmetric.

17. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Ans: Given relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$

Now, Reflexivity: Let $a \in A$

We have, $a \neq a + 1 \Rightarrow (a, a) \notin R$

\therefore It is not reflexive

Symmetric: Let $a = 1$ and $b = 2$ i.e. $a, b \in A$

$\therefore b = a + 1 \Rightarrow 2 = 1 + 1 \Rightarrow (a, b) \in R$

but $a \neq b + 1$ as $1 \neq 2 + 1 \Rightarrow (b, a) \notin R$

\therefore It is not symmetric.

Transitive: Let $a, b, c \in A$

Now, if $(a, b) \in R \Rightarrow b = a + 1 \dots(i)$

and $(b, c) \in R \Rightarrow c = b + 1 \dots(ii)$

from (i) and (ii), we have

$c = (a + 1) + 1 = a + 2$

$\Rightarrow c = a + 2 \Rightarrow (a, c) \notin R$

\therefore It is not transitive

Hence, relation R is neither reflexive nor symmetric nor transitive.

SECTION – D

Questions 18 carry 5 marks.

18. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

Ans: $A = \{x \in Z : 0 \leq x \leq 12\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and

$R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

For any element $a \in A$, we have $(a, a) \in R \Rightarrow |a - a| = 0$ is a multiple of 4.

$\therefore R$ is reflexive.

Now, let $(a, b) \in R \Rightarrow |a - b|$ is a multiple of 4.

$\Rightarrow |-(a - b)|$ is a multiple of 4

$\Rightarrow |b - a|$ is a multiple of 4.

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric.

Now, let $(a, b), (b, c) \in R$.

CD SIR (Chandra Dev Singh)

Founder , Mentor , Subject Expert
& Career Counsellor at CBSE ACADEMY PLUS

SURYADEV SINGH (SURYA BHAIYA)

Data Scientist, IIT Guwahati | M.Sc (IIT Delhi)
Director & Educator at CBSE Academy Plus

CBSE ACADEMY PLUS

$\Rightarrow |a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4.

$\Rightarrow (a - b)$ is a multiple of 4 and $(b - c)$ is a multiple of 4.

$\Rightarrow (a - b + b - c)$ is a multiple of 4

$\Rightarrow (a - c)$ is a multiple of 4

$\Rightarrow |a - c|$ is a multiple of 4

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is $\{1, 5, 9\}$ since

$|1 - 1| = 0$ is a multiple of 4

$|5 - 1| = 4$ is a multiple of 4

$|9 - 1| = 8$ is a multiple of 4

OR

Show that the function $f: R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one and onto function.

Ans: It is given that $f: R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$

Suppose, $f(x) = f(y)$, where $x, y \in R \Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$

It can be observed that if x is positive and y is negative, then we have

$$\frac{x}{1+x} = \frac{y}{1-y} \Rightarrow 2xy = x - y$$

Since, x is positive and y is negative, then $x > y \Rightarrow x - y > 0$

But, $2xy$ is negative. Then, $2xy \neq x - y$.

Thus, the case of x being positive and y being negative can be ruled out.

Under a similar argument, x being negative and y being positive can also be ruled out. Therefore, x and y have to be either positive or negative.

When x and y are both positive, we have $f(x) = f(y) \Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$

When x and y are both negative, we have $f(x) = f(y) \Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x - xy = y - xy \Rightarrow x = y$

Therefore, f is one-one. Now, let $y \in R$ such that $-1 < y < 1$.

If y is negative, then there exists $x = \frac{y}{1+y} \in R$ such that

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1+\left|\frac{y}{1+y}\right|} = \frac{\frac{y}{1+y}}{1+\frac{-y}{1+y}} = \frac{y}{1+y-y} = y$$

If y is positive, then there exists $x = \frac{y}{1-y} \in R$ such that

$$f(x) = f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1+\left|\frac{y}{1-y}\right|} = \frac{\frac{y}{1-y}}{1+\frac{y}{1-y}} = \frac{y}{1-y+y} = y$$

Therefore, f is onto. Hence, f is one-one and onto.

CD SIR (Chandra Dev Singh)

Founder , Mentor , Subject Expert
& Career Counsellor at CBSE ACADEMY PLUS

SURYADEV SINGH (SURYA BHAIYA)

Data Scientist, IIT Guwahati | M.Sc (IIT Delhi)
Director & Educator at CBSE Academy Plus

CBSE ACADEMY PLUS

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. An organization conducted bike race under two different categories-Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions:

- (i) How many relations are possible from B to G ? (1)
(ii) Among all the possible relations from B to G, how many functions can be formed from B to G ? (1)
(iii) Let $R : B \rightarrow B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}$. Check if R is an equivalence relation. (2)

OR

- (iii) A function $f : B \rightarrow G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check iff is bijective. Justify your answer. (2)

Ans: (i) Number of relations = $2^{mn} = 2^{2 \times 3} = 2^6 = 64$

(ii) Number of functions from B to G = $2^3 = 8$

(iii) $R = \{(x, y) : x \text{ and } y \text{ are students of same sex.}\}$

Since x and x are of the same sex

So $(x, x) \in R$ for all x

$\therefore R$ is reflexive

If x and y are of the same sex then y and x are also of the same sex

$\therefore R$ is symmetric

If $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$

Then x and z will be of the same sex

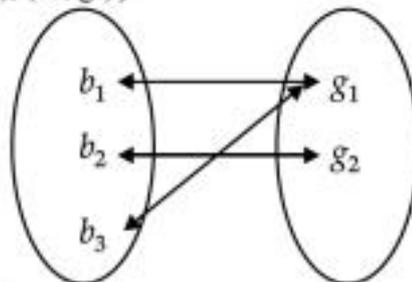
$\therefore R$ is transitive

Since R is reflexive, symmetric and transitive

$\therefore R$ is an equivalence relation.

OR

- (iii) Given $R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$



Since b_1 and b_3 have the same image g_1

$\therefore R$ is not injective

Since all elements of G has a pre-image

$\therefore R$ is bijective

CD SIR (Chandra Dev Singh)

Founder , Mentor , Subject Expert
& Career Counsellor at CBSE ACADEMY PLUS

SURYADEV SINGH (SURYA BHAIYA)

Data Scientist, IIT Guwahati | M.Sc (IIT Delhi)
Director & Educator at CBSE Academy Plus

CBSE ACADEMY PLUS

20. Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L .



(i) Let relation R be defined by $R = \{(L_1, L_2) : L_1 \parallel L_2 \text{ where } L_1, L_2 \in L\}$. What is the type of relation R ? (2)

(ii) (a) Check whether the function $f : R \rightarrow R$ defined by $f(x) = x - 4$ is bijective or not. (2)

OR

(ii) (b) Let $f : R \rightarrow R$ be defined by $f(x) = x + 4$. Find the range of $f(x)$. (2)

Ans: (i) Given relation R defined by

$$R = \{(L_1, L_2) : L_1 \parallel L_2 \text{ where } L_1, L_2 \in L\}$$

Reflexive: Let $L_1 \in L \Rightarrow L_1 \parallel L_1 \Rightarrow (L_1, L_1) \in R$.

\therefore It is reflexive.

Symmetric: Let $L_1, L_2 \in L$ and let $(L_1, L_2) \in R$

$$\Rightarrow L_1 \parallel L_2 \Rightarrow L_2 \parallel L_1 \Rightarrow (L_2, L_1) \in R$$

\therefore It is symmetric.

Transitive: Let $L_1, L_2 \in L$ and let $(L_1, L_2) \in R$

$$\Rightarrow L_1 \parallel L_2$$

Let $L_2, L_3 \in L$ and let $(L_2, L_3) \in R$

$$\Rightarrow L_2 \parallel L_3$$

Now, $L_1 \parallel L_2$ and $L_2 \parallel L_3 \Rightarrow L_1 \parallel L_3 \Rightarrow (L_1, L_3) \in R$

\therefore It is transitive.

Hence R is an equivalence relation.

(ii) (a) Given function $f : R \rightarrow R$ defined by $f(x) = x - 4$

Injective : Let $x_1, x_2 \in R$ such that $x_1 \neq x_2$.

$$\Rightarrow x_1 - 4 \neq x_2 - 4 = f(x_1) \neq f(x_2)$$

\therefore It is injective.

Surjective : Let $y = x - 4 \Rightarrow x = y + 4$

For every $y \in R$ (co-domain) there exists $x = y + 4 \in R$ (domain).

i.e., Co-domain = Range

\therefore It is surjective.

Hence given function is bijective.

OR

(ii) (b) Given function $f : R \rightarrow R$ defined by $f(x) = x + 4$

$$\text{Let } y = f(x) \Rightarrow y = x + 4 \Rightarrow x = y - 4$$

For $y \in R$ (co-domain),

$$\exists x = y - 4 \in R \text{ (domain) such that } f(x) = B$$

\therefore Range of $f(x)$ is R (Set of real numbers).